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Finite Amplitude Waves on Aircraft Trailing Vortices

by

D. W. Moore

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FINITE AMPLITUDE WAVES ON AIRCRAFT TRAILING VORTICES

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Abstract

Numerical methods are used to study the growth of waves of finite amplitude on a pair of parallel infinite vortices. The vortices are treated as lines except in so far as the detailed structure of the core is needed to remove consistently the singularity in the line integrals for the velocities of the vortices. It is shown that the vortices eventually touch and the shape of the wave at this instant is calculated. The wave is quite distorted at this instant, but it is shown that its gross properties are given roughly by linear theory.

§1. Introduction

The trailing vortex system behind a large aircraft contains a considerable amount of energy and presents a hazard to other, particularly smaller, aircraft. The recognition of this problem has lead to attempts, both theoretical and experimental, to estimate the life-time of the trailing vortex system.

To do this theoretically one must isolate the mechanism which is responsible for destroying the vortex trail. Turbulent diffusion of the vorticity is too gradual to matter and only two processes seem able to cause rapid disintegration of the vortex system, vortex bursting (Bisgood et al 1971) and the growth of waves of large amplitude.† This latter effect can often be observed when the vortex system of a high-flying aircraft is rendered visible by condensation. Wavy disturbances grow on both trailing vortices and rapidly reach an amplitude such that the two vortices touch at the points of maximum inward displacement. The trail then breaks up into a sequence of distinct rings and soon ceases to be visible. Several systematic observational studies of this "looping" process have recently been made (Rose and Dee 1963, Crow and Murman 1970, Olsen 1971, Widnall, Bliss and Zalay 1971).

It is worth emphasizing that the mechanism by which the original vortices break and rejoin to form rings is not understood. Nor is it clear why the vortex rings, which are stable on classical theory, should disappear so rapidly (if indeed they do, since care must be taken in

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†At high altitudes large stable density gradients can occur, and Scorer and Davenport (1970) have suggested that buoyancy effects can then lead to rapid disintegration.

interpreting observations which depend on the retention by the vortex of smoke particles or water droplets). However, the present paper is concerned only with the motion of the vortices up to the instant at which they touch.

Theoretical work on the growth of waves was initiated by Crow (1970). Crow represented the trailing vortex system as a pair of infinite parallel vortex filaments of circulation $\pm\ \Gamma$ having circular cross sections of radii a small compared to their separation b. Small perturbations in the form of plane waves were shown to be unstable, provided only that the waves were sufficiently long, the time of growth being of order b^2/Γ. In the mode of most rapid growth the disturbances lie in planes inclined at about 45° to the vertical which are oriented so as to meet below the undisturbed positions of the vortices and the waves themselves are symmetric in the sense that the crests of the wave on the left-hand vortex are directly opposite the crests of the waves on the right-hand vortex. The wavelength of this mode of maximum growth rate depends weakly on a/b and, for the value taken from the simplified theory of Spreiter and Sacks (1951), Crow showed that $\lambda/b = 8.5$.

According to linear theory the troughs of the waves will descend in their respective planes until they meet, so that an explanation of the first stage of the looping process is to hand. However the waves are of amplitude comparable to their length when they touch, so that it is clearly desirable to examine non-linear effects.

A first attempt has been reported by Hackett and Theisen (1971) and it was shown that while the wave is modified by non-linear effects, the troughs do eventually meet in the manner suggested by linear theory.

However their numerical method did not permit the effect of axial flow in the vortex filaments or differing filament radii to be studied and it was thought worthwhile to present a numerical method in which these effects could be allowed for.

The assumptions made are precisely those of Crow's theory. The fluid is regarded as inviscid and incompressible and has uniform density, while the vortex system is modelled as described above. To simplify the calculation still further, the vortices are treated as being of zero cross section and the velocity field due to either vortex is calculated from the Biot-Savart line integral. The neglect of viscous effects means that these vortex lines move with the fluid, and this leads to a pair of coupled integro-differential equations for the equations of the vortex lines.

A difficulty occurs because the Biot-Savart integrals diverge on the vortex lines themselves, but this can be dealt with by means of a "cut-off." The choice of this cut-off depends on the detailed structure of the vortex filament, particularly its radius and the radial distribution of vorticity and axial flow. It is only through the cut-off that these details influence the calculation.

These matters are described more fully in §2 and in §3 some transformations required before efficient numerical integration is possible are described.

In §4 the results of numerical integration are described for the case already discussed on linear theory by Crow. There is no axial flow and the vorticity is uniform inside the filament, which has the radius given by the Spreiter and Sacks theory. Initial plane waves of small amplitude and wavelength and orientation corresponding to Crow's

most unstable mode are prescribed. The subsequent growth of the disturbance is then followed by step-wise integration of the integro-differential equations.

The wave remains almost plane, but is somewhat distorted from a sinusoidal form by the time the troughs meet.

In view of the lack of reliable information about the distribution of vorticity and axial velocity in the filament in the real case it was not thought to be worthwhile to carry out further calculations at present, though the program can deal with the general case.

§2. The equations of motion

Rectangular axes moving with the undisturbed vortices are employed, Oz being parallel to the vortices and midway between them and Ox being perpendicular to the plane defined by the undisturbed positions of the vortices. For an aircraft flying horizontally Oz will be in the direction of flight and Ox will be vertically upwards.

The vortices will be represented parametrically as $\underset{\sim}{x}_L(v,t)$ and $\underset{\sim}{x}_R(v,t)$ where v is a parameter chosen so that v = const. represents always the same fluid particle. Then, bearing in mind that the fluid at large distances has a velocity $\frac{\Gamma}{2\pi b}\,\underset{\sim}{i}$ relative to the moving axes, the equation of motion of the left-hand vortex is

$$\left(\frac{\partial \underset{\sim}{x}_L}{\partial t}\right)_{v_0} = \frac{\Gamma}{4\pi} \oint_{-\infty}^{\infty} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right) dv}{|x_L(v_0)-x_L(v)|^3}$$

$$- \frac{\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\partial \underset{\sim}{x}_R}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_R(v)\right) dv}{|\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_R(v)|^3} + \frac{\Gamma \underset{\sim}{i}}{2\pi b}\,, \qquad (2.1)$$

and that of the right-hand vortex is

$$\left(\frac{\partial \underset{\sim}{x}_R}{\partial t}\right)_{v_1} = \frac{\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_R(v_1) - \underset{\sim}{x}_L(v)\right) dv}{|\underset{\sim}{x}_R(v_1) - \underset{\sim}{x}_L(v)|^3}$$

$$- \frac{\Gamma}{4\pi} \oint_{-\infty}^{\infty} \frac{\partial \underset{\sim}{x}_R}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_R(v_1)-\underset{\sim}{x}_R(v)\right) dv}{|\underset{\sim}{x}_R(v_1)-\underset{\sim}{x}_R(v)|^3} + \frac{\Gamma \underset{\sim}{i}}{2\pi b}\,. \qquad (2.2)$$

The notation $\oint$ implies that some artifice is used to make the integrals representing the self induction finite at $v = v_0$ or $v = v_1$.

Two methods of dealing with the divergence of the integral for the self-induced velocity have been employed. The simplest method and that used by Crow himself is to remove an interval $(v_0 - \epsilon,\ v_0 + \epsilon)$ from the range of integration, where the small quantity ϵ is chosen so that the arc length of the portion of the vortex corresponding to $v_0 - \epsilon < v < v_0 + \epsilon$ bears a fixed ratio $2\delta_c$ to the core radius $a(v_0)$ at the point in question. The quantity δ_c is chosen so that the velocity given by the "cut-off" integral agrees with that given by the exact solution for a circular vortex. Remarkably this choice of δ_c can be shown to give the correct self-induced velocity for a vortex of any shape provided that terms of $O(\frac{a}{\rho})$ are neglected, where ρ is the radius of curvature of the vortex (Widnall, Bliss and Zalay 1971, Moore and Saffman 1971).

This method of cut-off is not flexible enough for numerical work, because it would be awkward to remove an interval from the line integral which was not terminated by grid points of the finite difference scheme. It may be remarked at this point that dealing with the infinity by simply omitting the grid point at which the integral is infinite, as was done by Hackett and Theisen (1971), is equivalent to a choice of core radius dictated by the mesh-size of the finite difference scheme.

A suitable method of cut-off is that suggested by Rosenhead (1930), and consists of replacing the denominator in the integral for the self induced velocity in (2.1) by $\left|\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right)^2 + \mu^2\right|^{3/2}$ where the small length μ bears a fixed ratio $2\delta_R$ to the core radius $a(v_0)$ and similarly for (2.2). Rosenhead merely remarked that μ is $O(a)$ but by comparing the solution using a cut-off integral with the exact solution for a ring, as generalized by Widnall et al (1971), it can be shown that

$$\log 2\delta_R = -\frac{1}{2} - \frac{1}{\Gamma^2(a)}\int_0^a \Gamma^2(s)\,\frac{ds}{s} + \frac{8\pi^2}{\Gamma^2(a)}\int_0^a sW^2\,ds \,, \qquad (2.3)$$

where $\Gamma(s)$ is the circulation and $W(s)$ the axial velocity at a distance s from the centre of the vortex filament. For $W = 0$ and $\Gamma(s) = s^2/a^2$, so that there is no axial flow and uniform vorticity inside the filament (2.3) reduces to

$$\log 2\delta_R = -\frac{3}{4} \qquad (2.4)$$

For the reasons described in §1 this is the only case considered in detail. However one is now faced by a more difficult problem, which is how the core radius itself is to be determined, since until $a(v_0)$ is known μ cannot be determined. The vortex filaments will stretch as the waves grow so that, since their volume is conserved, their cross-sectional area will certainly have to change.

This question has been studied elsewhere (Moore and Saffman 1971) and it has been shown that a is sensibly uniform along the vortex, being just that function of time only which conserves the volume of the filament. Essentially this is due to the fact that changes of cross-sectional area propagate along the vortex with speed $O(\frac{\Gamma}{a})$, whereas changes of curvature or torsion travel with speed $O\left(\frac{\Gamma}{\rho}\log\frac{\rho}{a}\right)$. Thus variations of cross-sectional area are smoothed out in a time short compared to the times of interest here.

This is a great simplification and means that the radius of the filament is easily determined from the instantaneous length of the filament.

It is now a simple matter in principle to determine the evalution of any initial disturbance $\mathbf{x}_L(v, 0)$, $\mathbf{x}_R(v, 0)$ to the vortex pair, by integrating (2.1) and (2.2) forward in time, the length of the filaments being calculated at each time step in order to obtain the appropriate value of μ. However some transformations of (2.1) and (2.2) which take account of the periodic nature of the disturbance greatly decrease the amount of computing necessary and these are described in the next section.

§3. Reduction to a form suitable for numerical integration

Only symmetric disturbances are to be considered so that

$$\left.\begin{aligned} x_L(v,t) &= x_R(v,t) \,, \\ y_L(v,t) &= -y_R(v,t) \,, \\ z_L(v,t) &= z_R(v,t) \,. \end{aligned}\right\} \tag{3.1}$$

Hence only (2.1) need be integrated and (2.2) is replaced by the symmetry conditions embodied in (3.1).

Next advantage is taken of the fact the disturbances are periodic, which means that the parameter v can be chosen so that for any integer n

$$\underset{\sim}{x}_L(v+n,t) = \underset{\sim}{k}\, n\lambda + \underset{\sim}{x}_L(v,t) \,. \tag{3.2}$$

Here, λ is the wavelength of the disturbance, which does not change as the wave grows, and v has been chosen so that $0 \leqslant v \leqslant 1$ corresponds to one wavelength.

Thus one need only follow, say, the portion $-\frac{1}{2} \leqslant v \leqslant \frac{1}{2}$ to obtain $\underset{\sim}{x}_L(v,t)$ and hence, by (3.1), $\underset{\sim}{x}_R$. But the integrals in (2.1) converge rather slowly and have large integrands, so that further steps are needed to make accurate numerical integration feasible.

The first integral in (2.1) can be decomposed into the form

$$\oint_{-1}^{+1} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right)dv}{|\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)|^3} + \sum_{m=-\infty}^{m=\infty,} \oint_{-1}^{+1} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)-2m\underset{\sim}{k}\lambda\right)dv}{|\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)-2m\underset{\sim}{k}\lambda|^3} \tag{3.3}$$

where the second term arises from summing over successive portions of the disturbance of length 2λ. If v_0 is restricted to lie in the interval $(-\frac{1}{2}, \frac{1}{2})$, all the integrands in the second term are bounded † and no cut-off is required. Thus the integral can be written as

$$\oint_{-1}^{+1} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\underset{\sim}{\Delta}(v, v_0) dv}{|\underset{\sim}{\Delta}(v, v_0)|^3} + \int_{-1}^{+1} \frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \left(\underset{\sim}{\Delta}(v, v_0) P(\underset{\sim}{\Delta}) - \underset{\sim}{k} Q(\underset{\sim}{\Delta})\right) dv \qquad (3.4)$$

where

$$\underset{\sim}{\Delta}(v, v_0) = \underset{\sim}{x}_L(v_0) - \underset{\sim}{x}_L(v) \qquad (3.5)$$

and

$$\left.\begin{aligned} P(\underset{\sim}{u}) &= \sum_{m=-\infty}^{m=+\infty}{}' \left\{\underset{\sim}{u}^2 - 4m\,\underset{\sim}{u} \cdot \underset{\sim}{k} + 4m^2\right\}^{-\frac{3}{2}} \\ Q(\underset{\sim}{u}) &= \sum_{m=-\infty}^{m=+\infty}{}' 2m\left\{\underset{\sim}{u}^2 - 4m\,\underset{\sim}{u} \cdot \underset{\sim}{k} + 4m^2\right\}^{-\frac{3}{2}} \end{aligned}\right\} \qquad (3.6)$$

The sums of the infinite series in (3.6) do not seem to be expressible in terms of elementary functions, but they can be computed numerically. In practice a table of values over a discrete set of $\underset{\sim}{u}^2$ and $\underset{\sim}{u} \cdot \underset{\sim}{k}$ was computed and stored before time integrations were started, the values of P and Q being calculated for the actual $\underset{\sim}{u}$ arising in the time integration by bi-variate interpolation.

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†This is why the summation is over two waves at a time. I am indebted to Professor P. G. Saffman for this step.

This use of the periodicity, which was suggested by Rosenheads (1930) treatment of the two-dimensional vortex sheet, means that the range of integration is reduced from $(-\infty,\infty)$ to $(-1,1)$ so that high accuracy can be achieved in the numerical integration with a tractable number of grid points.

The second integral in (2.1) can be similarly transformed and the same functions P and Q arise.

However the integral in the first term in (3.4), though everywhere finite, is large near v_0 and this would cause loss of accuracy in evaluating the integral. It is easy to show that, when $v \sim v_0$,

$$\frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)}{\left\{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right)^2+\mu^2\right\}^{3/2}} \sim \frac{\frac{1}{2}(v-v_0)^2\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0 \wedge \left(\frac{\partial^2 \underset{\sim}{x}_L}{\partial v^2}\right)_0}{\left\{(v-v_0)^2\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)^2+\mu^2\right\}^{3/2}} \tag{3.7}$$

so that if the first term in (3.4) is written in the form

$$\int_{-1}^{+1}\left(\frac{\partial \underset{\sim}{x}_L}{\partial v} \wedge \frac{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right)}{\left\{\left(\underset{\sim}{x}_L(v_0)-\underset{\sim}{x}_L(v)\right)^2+\mu^2\right\}^{3/2}} - \frac{\frac{1}{2}(v-v_0)^2\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0 \wedge \left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0}{\left\{(v-v_0)^2\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0^2+\mu^2\right\}^{3/2}}\right) dv$$

$$+\frac{1}{2}\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0 \wedge \left(\frac{\partial^2 \underset{\sim}{x}_L}{\partial v^2}\right)_0 \int_{-1}^{+1} \frac{(v-v_0)^2}{\left\{(v-v_0)^2\left(\frac{\partial \underset{\sim}{x}_L}{\partial v}\right)_0^2+\mu^2\right\}^{3/2}}\, dv \tag{3.8}$$

then the integrand in the first integral is O(1) everywhere, while the second integral is elementary.

Finally it may be remarked that, in view of the uniformity of the cross section and the conservation of volume,

$$\mu = 2\delta_R a_0 \left\{ \frac{1}{\lambda} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left| \frac{\partial \underset{\sim}{x}_L}{\partial v} \right| dv \right\}^{-\frac{1}{2}} , \tag{3.9}$$

where a_0 is the uniform radius of the vortex filaments in the undisturbed state.

It is now a straightforward matter to follow numerically the evolution of a prescribed initial disturbance. The interval (-1,1) was divided into 2N portions by 2N+1 equally spaced grid points. The spatial derivatives were calculated using four-point centered differences and Simpson's rule was used to evaluate the integrals.

The integration forward in time was effected by the fourth-order Runge-Kutta formula, this method being used because of its stability.

Clearly dimensionless variables are preferable in the numerical work and variables

$$\underset{\sim}{x}' = \underset{\sim}{x}_L/\lambda$$

and (3.10)

$$t' = t\left(\Gamma/4\pi\lambda^2\right)$$

were found to be most convenient.

The results of a typical calculation are described in §4.

§4. Numerical Results

Crow (1970) has shown that the natural time scale for the growth of waves is $2\pi b^2/\Gamma$, which is just the time taken for the undisturbed vortices to descend a distance equal to their separation. Thus to present the results a dimensionless time t^* defined by

$$t^* = \frac{\Gamma}{2\pi b^2} t \tag{4.1}$$

will be employed.

The trailing vortices are assumed to have the radius given by Spreiter and Sacks (1951) so that

$$a/b = .098 \tag{4.2}$$

It must be stressed that it is far from certain that this is realistic for real trailing vortices (see, for example McCormick, Tangler and Sherrieb 1968) but as no firm value has emerged from experiment it seemed best to adhere to (4.2). In any case the results are not very sensitively dependent on a/b.

Crow's linear theory shows that for this value of a/b , the most unstable mode has wavelength 8.5b and lies in a plane inclined at 47.5° to the horizontal. It was decided to study the evolution of this mode from an initial semi-amplitude of 0.05b.

Trial and error showed that a time step $\Delta t^* = 0.1$ gave adequate accuracy and the value N = 40 (that is 40 points per wave) was used. However at $t^* = 2.4$ numerical instability sets in at the troughs of the waves,

presumably because the distance between the grid points was not small compared to the separation of the vortices and because the curvature of the wave is largest here. To cure this and to cater for the more rapid changes consequent on the proximity of the troughs the calculation was stopped at $t^* = 2.2$ and restarted with $N = 80$ and $\Delta t^* = 0.25$. This removed the instability.

At $t^* = 2.475$ the troughs of the vortices were only .1876 apart, which implies that the two vortices are touching. Clearly the results of a calculation which treats the separation of the vortices as large compared to their radii must be viewed with skepticism at this stage. However, a separate numerical study of the motion of cylindrical vortices in two dimensions in which core size was allowed for showed that the Biot-Savart formula gives roughly the correct velocity even when the cores are touching. Thus, while the cores are very distorted, which would affect the internal structure and thus change the cut-off, it is possible that the approximations on which the present paper is based are adequate even when the cores are close.

In any case, there is no point in integrating beyond the approximate instant of touching so the calculation was terminated at $t^* = 2.475$.

Figure 1 shows $\log_{10}\left[B(t^*)/B(0)\right]$ as a function of t^*, where

$$B = \frac{y_{max} - y_{min}}{y_{max} + y_{min}} \tag{4.2}$$

is a convenient measure† (introduced by Crow and Murman 1970) of the

†This quantity can be directly measured from photographs taken from vertically below the trail.

growth of the waves. According to Crow's linear theory

$$\log_{10}\left[B(t^*)/B(0)\right] = .368\,t^* \tag{4.3}$$

and this straight line is shown for comparison. The agreement is excellent for small times and indeed linear theory is adequate over nearly the whole range of values of t^*.

Figure 2 shows a plan view of the centre-lines of the vortices at the approximate instant of touching $t^* = 2.475$, while Figure 3 shows a side elevation at that time. For the reasons given, it is hard to assess the relevance to the actual situation of these results, but there is some similarity to observed large amplitude waves in that the curvature is longer at the trough than at the crest. The development of large curvature in the trough suggests that even more grid points would have been desirable the final stages of the calculation, but in view of the dubious relevance of this part of the calculation such a refinement was not attempted.

The end elevation is shown in Figure 4 and reveals that the wave remains practically plane, though the plane is not quite the original one. The increase of length of the wave was found to be only 5% at $t^* = 2.475$. The vortex stretches most at the trough where the distance between two fluid particles initially a distance δz apart increases to $1.6\,\delta z$ at the instant of touching. The vortex contracts at the crest where the distance between two fluid particles initially a distance δz about decreases to $0.9\,\delta z$. The differences reflect the marked asymmetry between trough and crest.

Acknowledgements. The author has benefited greatly from discussing this work with Professor P. G. Saffman.

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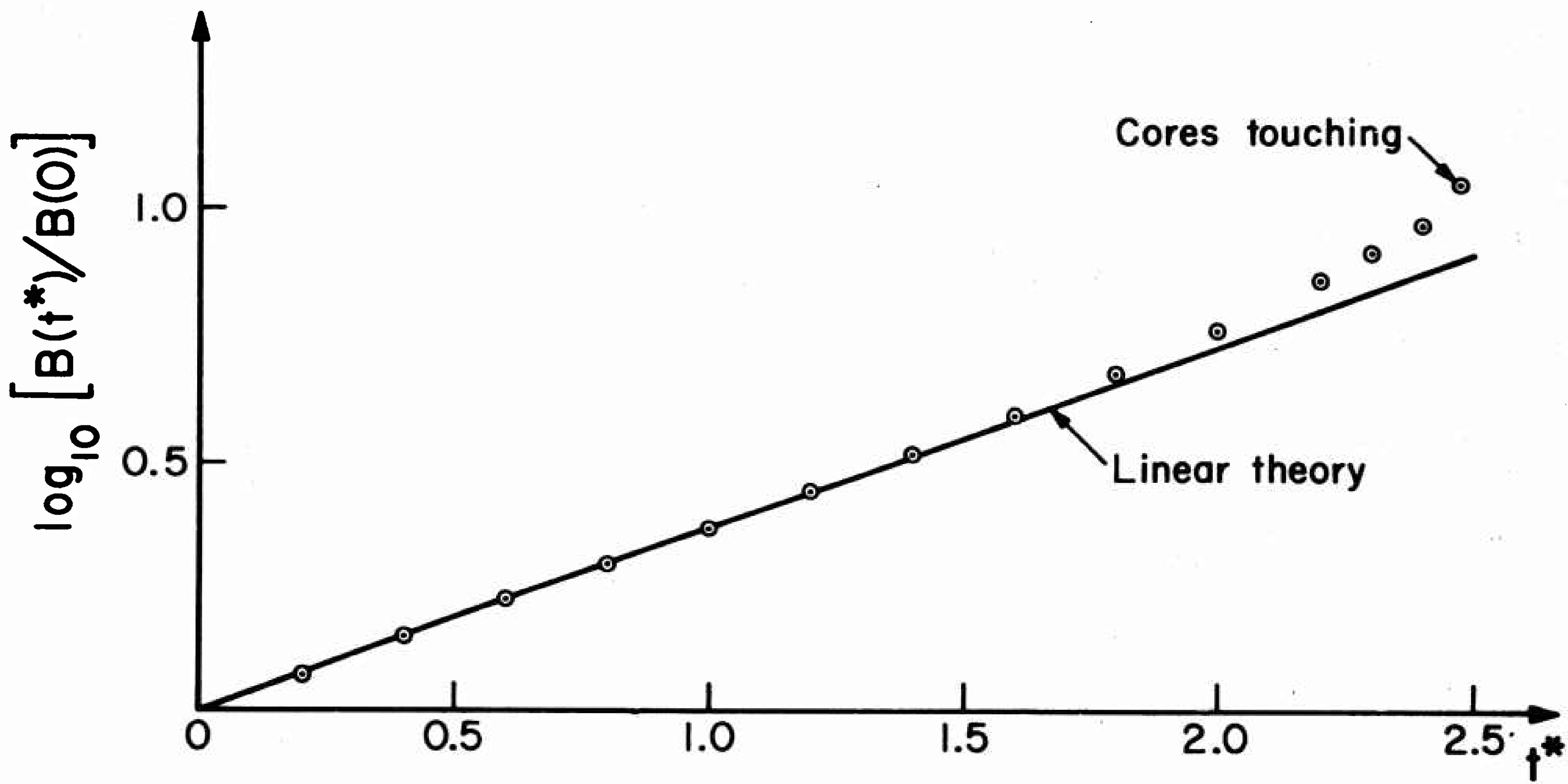
Figure Captions

Figure 1. The function $\log_{10}\left[B(t^*)/B(0)\right]$ as a function of t^* obtained from linear theory and from the numerical solution (circled points).

Figure 2. The plan view of the wave at $t^* = 2.475$, the approximate instant of touching. The initial core diameter is shown on the same scale.

Figure 3. The side elevation of the wave at $t^* = 2.475$.

Figure 4. The end elevation of the wave at $t^* = 2.475$.



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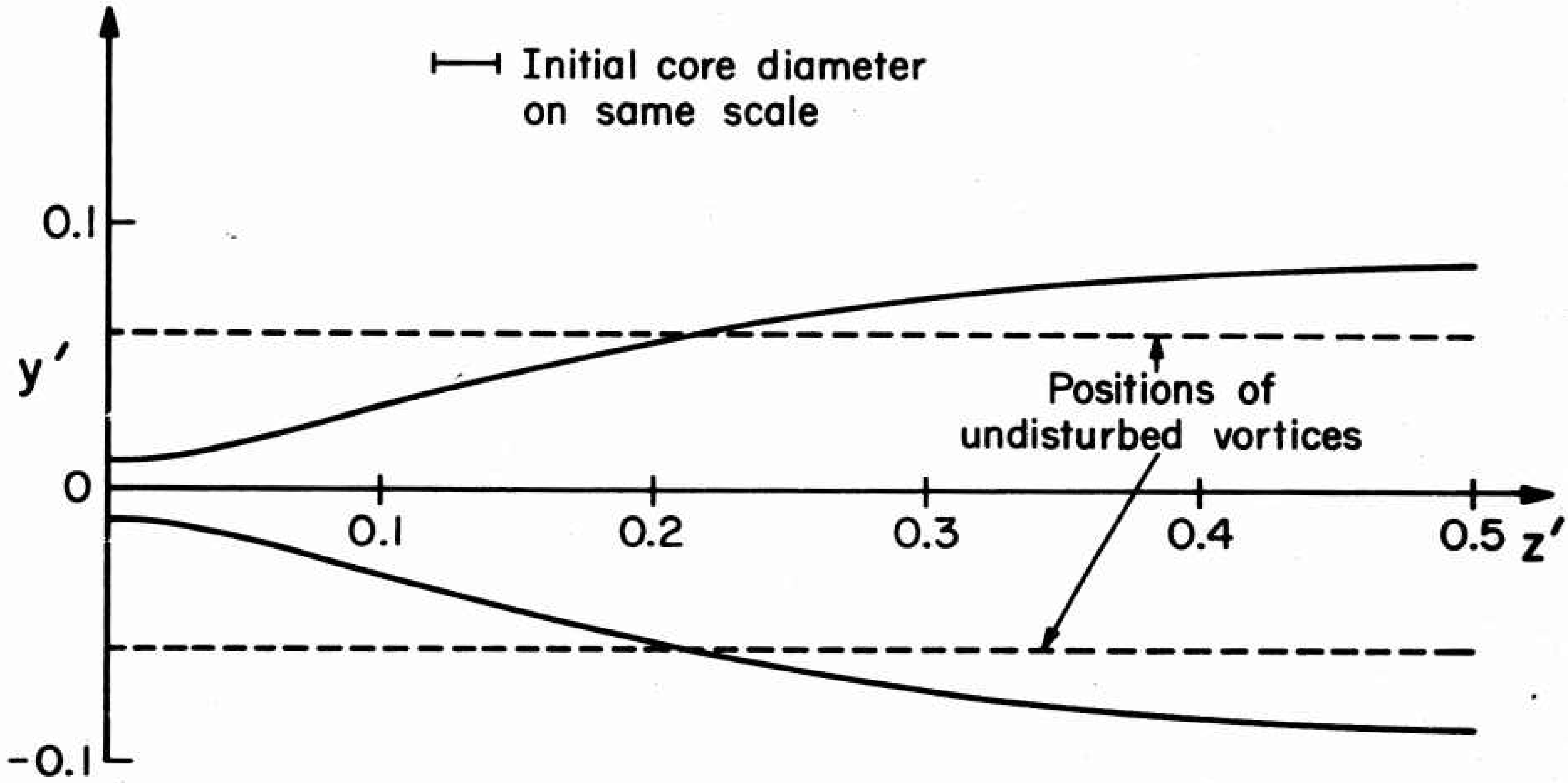

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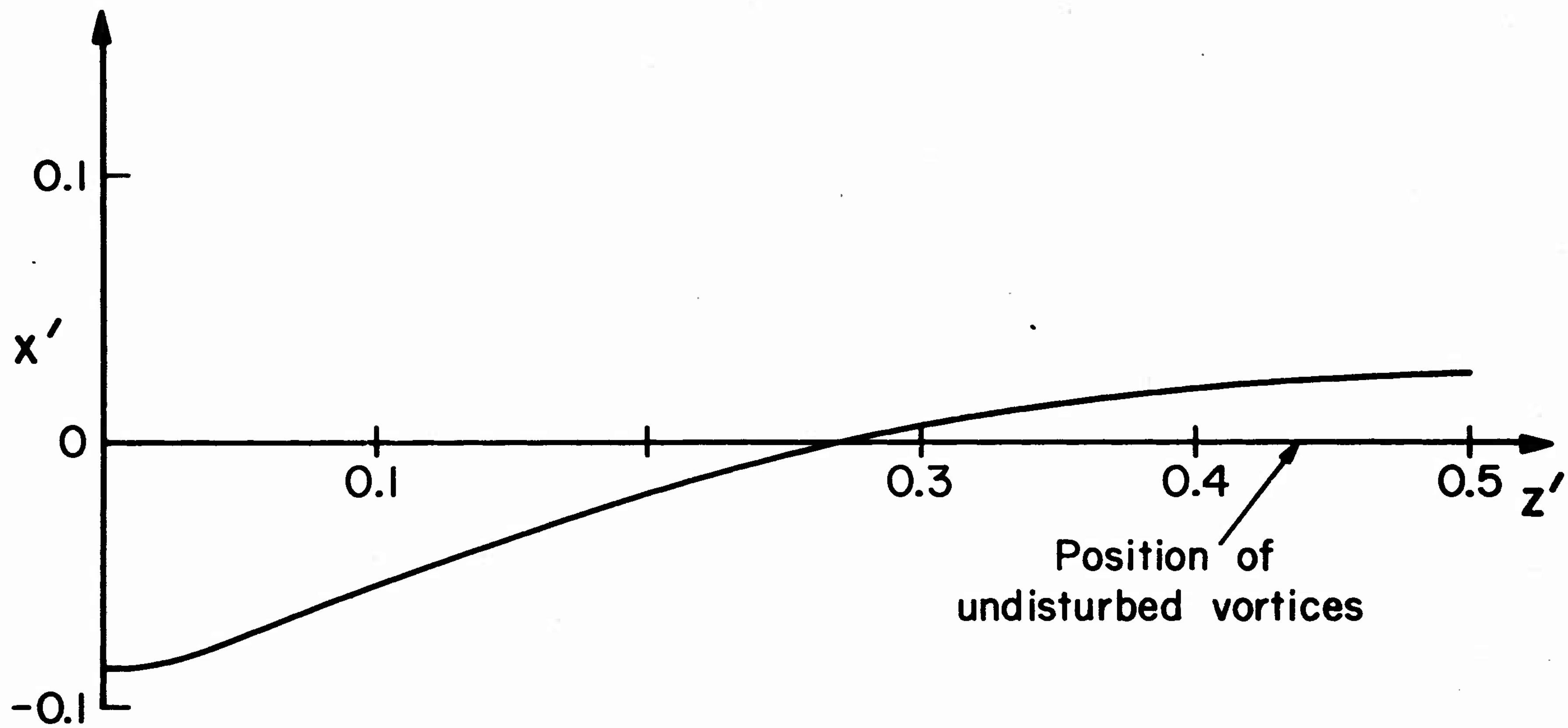

Figure 3. The side elevation of the wave at $t^* = 2.475$.

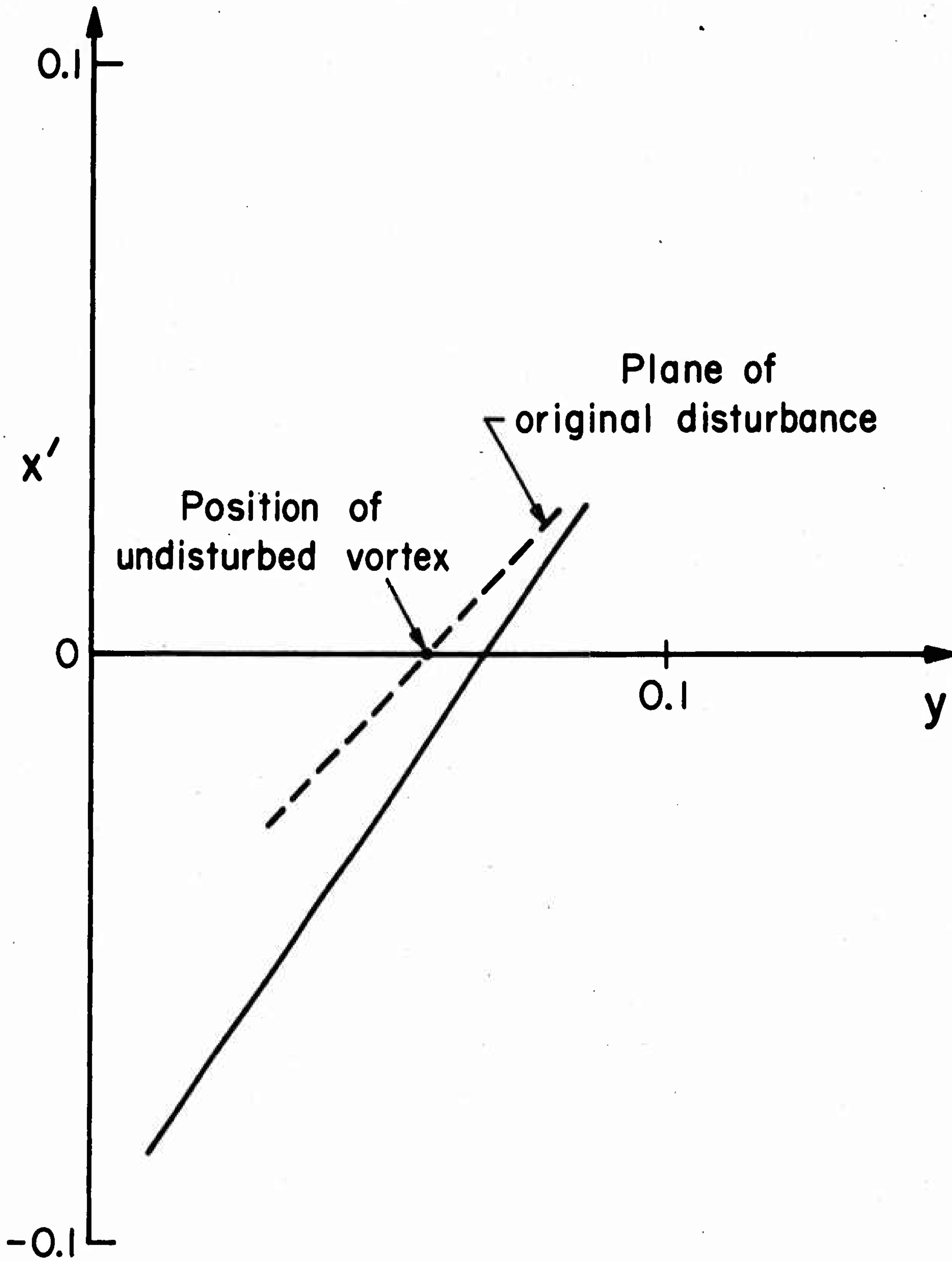

Figure 4. The end elevation of the wave at $t^* = 2.475$.